PREDICTION OF THE BEACH PROFILE OF HIGH DENSITY THICKENED TAILINGS FROM RHEOLOGICAL AND SMALL SCALE TRIAL DEPOSITION DATA

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Introduction

• Several approaches to beach prediction:
  – Classical hydrodynamics
  – Empirical
  – Stream Power

• This paper focuses on the stream power approach
Basis of the stream power approach

• Particles need energy to remain in suspension
• The energy per unit time consumed by particles in remaining in suspension is stream power
• At any point along the beach the stream power is a constant if:
  – The flow rate is constant
  – The density is constant
First, some equations

- Stream power is calculated from

\[ P = \rho g Q H \]

- In a loose boundary situation the stream power at a distance \( x \) is given by:

\[ P_x = \rho Q g \left( \frac{v_x^2}{2g} \right) = \rho Q \frac{v_x^2}{2} \]
Some logic

- The beach will be parallel to the energy line.
- If we can estimate the slope of the energy line at any point along the beach we can get the beach slope.
- If we aggregate the slope we get the profile.
- For a constant flow rate the slope of the stream power curve will be the slope of the energy line.
How can we calculate the slope of the stream power curve?
EEK! Entropy

• Entropy is a measure of randomness or probabilistic uncertainty of a variable.

• All probability distributions may be derived through an entropy-based approach.

• A great number of natural phenomena may be described using entropy.

• Maximisation/minimisation of entropy guarantees minimum bias.
EEK! Entropy

• Applied to stream power it means that of all the ways stream power can reduce with distance along the beach there is one that is most probable.

• McPhail demonstrated that entropy applied to stream power yields good predictions of the beach profile – for conventional tails.
So…

- Will entropy-stream power work for high density thickened tails?
- How could it be applied?
Some more equations

• Applying entropy maximisation to stream power yields the following equation:

\[ P(x) = -\frac{1}{\mu} \ln \left[ (1 - \exp^{-\mu P_0}) \frac{x}{L} + \exp^{-\mu P_0} \right] \]

• Differentiating gives the slope at any point on the beach:

\[ S_B(x) = -\frac{(1 - \exp^{-\mu P_0})}{L \mu \exp^{-\mu P(x)}} \]
Some more equations

• The elevation of the beach, $y$, above the datum is given by:

$$y = y(x + \Delta x) + S_B(x)\Delta x$$

• The slope at the start of the beach is given by

$$S_0 = -\frac{(1 - \exp^{-\mu P_0})}{L \mu \exp^{-\mu P_0}}$$
Information needed

- The elevation of the beach at the start of the beach profile
- The length of the beach
- Slurry density
- Slurry flow rate
- An estimate of the stream power at the start of the beach
- An estimate of the initial slope of the beach.
Stream power at the start of the beach

- Not equal to the stream power in the pipe
- Determined by the plunge pool formed when the slurry strikes the beach
Plunge pool

Trajectory

Plunge Pool

Metago Environmental Engineers
Examples
Plunge pool calculations

• Standard hydraulics calculations for hydraulic jumps using specific energy and change in momentum

• Energy loss highly dependent on:
  – Flow rate
  – Velocity head at end of pipe
  – Height of pipe above beach
  – Slurry density
Estimation of the initial slope of the beach

- Need friction flow characteristics of the slurry
- Establish these from rheological testing
Homogeneous non-settling slurry rheogram

Shear stress

Increasing slurry density

Shear rate
Non-homogeneous settling slurry

- Standard lab viscosity testing on a settling slurry yields spurious results due to settling out in the viscometer at low shear rates
- Most hard rock mining tailings produce non-homogeneous, settling slurries
- Beaching occurs at low shear rates
Settling slurry viscometer measurements

Irregular flow curves due to some sort of settling

Shear Stress, τ₀ (Pa)

Shear Rate, γ (s⁻¹)

81.4%m
78.9%m
77.7%m
72.1%m
Enter flume and small scale deposition testing

- Able to simulate deposition
- Relatively easy to vary density
- Problem - scaling up to full scale
Small scale testing as a rheometer

• Use stream power to fit beach profile and calibrate \( P_0 \), the stream power at the lip of the plunge pool by varying \( \mu \)
• Confirm this using plunge pool calculations
• Calculate the velocity at the lip of the plunge pool using:
  \[
  \frac{v^2}{2g} = \frac{P_0}{\rho g} = q \frac{v_0^2}{2}
  \]
• Calculate the shear rate of the slurry at the lip of the plunge pool using:
  \[\omega = 8v / R\]
Last of the equations

- Calculate the fanning friction coefficient using:

\[ S_B = \frac{4fv^2}{2gR} \]

- Calculate the shear stress in the slurry at the lip of the plunge pool using:

\[ \tau = \frac{fp\nu}{2} \]
Plot results

Shear stress

Increasing slurry density

Shear rate
Scale up to full scale

- Apply plunge pool calculations to determine $P_0$ and the shear rate
- Calculate $\mu$  
  
  \[
  P_0 = \frac{I}{\mu} \ln \left[ \exp^{-\mu P_0} \right]
  \]
- Determine the shear stress
- From this calculate $f$  
  
  \[
  \tau = f \rho v / 2
  \]
- Calculate the initial slope of the beach  
  \[
  S_0 = -\frac{(I - \exp^{-\mu P_0})}{L \mu \exp^{-\mu P_0}}
  \]
- Predict the beach profile for a given beach length  
  
  \[
  P(x) = -\frac{I}{\mu} \ln \left[ (1 - \exp^{-\mu P_0}) \frac{x}{L} + \exp^{-\mu P_0} \right]
  \]

\[
 y = y(x + \Delta x) + S_B(x) \Delta x
\]
Case study 1

- Solids SG 3.6, hard rock tailings
- Max particle dia: 1.18mm
- 50% passing 75μ
- Flume 7m long, 0.5m wide
- 25mm dia discharge pipe
- 0.8 l/s
Flume beaches

Flume test 68% solids

Flume test 68% solids repeat

Flume test 71% solids

Flume test 74% solids
Full scale beaches

**Full scale 55% solids**

**Full scale 66.5% solids**

**Full scale 68% solids**

**Full scale 74% solids**

**Full scale 77% solids**
Rheogram

- **Shear rate (8v/R) s^-1**
- **Shear stress (Pa)**

- **68% Flume**
- **68% Flume repeat**
- **71% Flume**
- **74% Flume**
- **55%**
- **68%**
- **70.0%**
- **74%**
- **77%**

- *Fitted Power Law* for 74% solids
- *Fitted Power Law* for 68% solids
Compare with lab measurements
Observations

• Good fits of beach profiles for
  – wide range of % solids, and
  – between flume and full scale
• Both figures are rheograms
• Both mirror the rise in shear stress as the shear rate decreases and settling occurs
More observations

• In this case the flume shear rates are significantly higher than the full scale
• More reliable predictions if shear rate in flume were close to that in the full scale.
Case study 2

- SG solids 2.7, hard rock tailings
- Max particle dia: 3.35mm
- 15.5% passing 75μ
- 12m by 12 m paddocks
- One sloping downwards, one upwards
- Two slurry concentrations:
  - 74% solids (1.8t/m³)
  - 78% solids (1.9t/m³)
Trial beach profiles

Pilot-scale 73.7% solids

Pilot scale 77.4% solids
Rheogram

Shear stress (Pa)

Shear rate (6v/d) s^-1

- 74%
- 77%
- Applied Power Law for 74% solids
Prediction for full scale operation

- Slurry density 1.8t/m³ (74% solids)
- \( P_0 = 19.7 \text{ Watts, shear rate } 6.3 \text{s}^{-1} \)
- From rheogram, shear stress = 201kPa
- Calculate:
  - Friction slope at lip of plunge pool = 0.031
  - Average beach slope over 500m = 1:26 (3.9%)
Conclusions

• Entropy stream power handles:
  – Conventional and high density
  – Flume and full scale

• For homogeneous, non-settling slurries lab-derived rheograms are acceptable

• For non-homogeneous, settling slurries use small scale trials which are effectively viscometers

• Small scale trials allow a range of slurry densities to be evaluated

• Reliability improves if the tests are set such that the shear rate is close to full scale

• Predictions are plausible
Finally, in a nutshell

• The entropy-stream power approach is versatile in predicting full scale beaching profiles from lab and small scale testing

• The only data required is:
  – The rheogram of the slurry
  – The full scale pipe size
  – Discharge conditions
  – Slurry density
  – Flow rate
  – Beach length
Thank you